DOCUMENT RESUME

ED 478 194 SE 068 160

AUTHOR	Shafer, Mary C.; Romberg, Thomas A.
TITLE	Assessment in Classrooms That Promote Understanding.
ISBN	ISBN-0-8058-3028-6
PUB DATE	1999-00-00
NOTE	27p.; In: Fennema, Elizabeth, Ed. and Thomas A. Romberg, Ed., Mathematics Classrooms That Promote Understanding, Mahwah, NJ, Lawrence Erlbaum, 1999. p159-84. For entire book, see SE 068 002.
AVAILABLE FROM	Lawrence Erlbaum Associates, Inc., 10 Industrial Avenue, Mahwah, NJ 07430.
PUB TYPE	Reports - Descriptive (141)
EDRS PRICE	EDRS Price MF01 Plus Postage. PC Not Available from EDRS
DESCRIPTORS	*Alternative Assessment; Educational Research; Elementary Secondary Education; *Evaluation; Learning; Mathematics Curriculum; *Mathematics Instruction

ABSTRACT

This chapter covers assessment issues in classrooms and offers a possible assessment program. The purpose is three-fold: (1) to examine ways of documenting students' understanding using a domain-based approach to assessment; (2) to present examples of assessment items related to this approach; and (3) to discuss difficulties that have arisen as teachers have implemented such an approach to assessment. (SOE)

Reproductions supplied by EDRS are the best that can be made

ED 478 194

IMISSION TO REPRODUCE AND
SSEMINATE THIS MATERIAL IN
MICROFICHE ONLY HAS BEEN GRANTED BY

S. Parker

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)
☐ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Assessment in Classrooms That Promote Understanding

Mary C. Shafer
Thomas A. Romberg
University of Wisconsin–Madison

Other chapters provide compelling evidence that teachers can and have used a domain-based approach to create classrooms that promote student understanding. Yet these teachers also must assess student understanding and performance as a consequence of instruction and be able to trace the growth of that understanding over time. Using a domain-based approach to the development of understanding requires a close look at the ways we can appropriately assess students and still provide sufficient evidence of student learning for parents, administrators, and the general public. Our purposes in this chapter are threefold: (a) to examine ways of documenting students' understanding, using a domain-based approach to assessment; (b) to describe examples of assessment items related to this approach; and (c) to discuss difficulties that have arisen as teachers have implemented such an approach to assessment.

DESIGNING AN APPROPRIATE ASSESSMENT PROGRAM

The difference between a domain-based approach to assessment and traditional assessment lies in the meaning attributed to understanding in a mathematical domain. Understanding the concepts of a domain involves more than accumulating a set of facts and procedures. Rather, understanding develops as new relationships among pieces of existing knowledge

BEST COPY AVAILABLE

are sought, tested, and realized, or as new information is connected to and integrated with existing knowledge. As understanding grows, the facts, relationships, and procedures in a domain become resources that aid reasoning in solving ordinary problems in routine ways and in generating insights for making sense of unfamiliar situations. This growth in understanding in a domain occurs intermittently and sporadically in periods of progression and regression, rather than in linear increments or stages. Assessment using a domain-based approach, therefore, is not limited to checking whether students have acquired mathematical facts and procedures emphasized during instruction. Assessment focuses on the new relationships that students are forming among mathematical ideas, their use of existing knowledge in new situations, and the levels of reasoning applied in solving problems. Assessment examines the ways students' knowledge is changing over time during instructional units related specifically to the domain, as well as in instructional units focused on other domains, during one grade level and across grade levels.

The ideas expressed in this chapter are based on other assumptions:

- Assessment should be viewed as an ongoing process that is integrated with instruction.
- Multiple sources of evidence are needed to assess students' developing knowledge in a domain.
- Assessment should involve the deliberate documentation of information derived from classroom interaction as well as from written work.

In this chapter we examine one assessment program using a domain-based approach to documenting student understanding in mathematics and point out the possibilities of its implementation. We focus on assessment of the growth in mathematical knowledge through attention to the mental activities that promote learning with understanding described in chapter 2 by Carpenter and Lehrer (i.e., constructing relationships, extending and applying mathematical knowledge, reflecting on mathematical experiences, articulating what one knows, making mathematical knowledge one's own) at varying levels of complexity.

Key Aspects of Design

Because learning mathematics with understanding develops gradually over time through active engagement in mathematical thinking, assessment using a domain-based approach requires changes in thinking about and planning assessment opportunities. Understanding the instructional sequence for the development of the concept throughout the curriculum is essential, especially in domain-based curricula designed to enhance gradual

formalization of mathematical concepts. In such curricula, understanding of formal mathematical symbols and procedures develops through students' informal reasoning strategies and the use of mathematical tools (such as the fraction bar) over several units in one grade level or across grade levels. Evidence of individual students' knowledge in a particular domain should, therefore, be gathered during all instructional units related to the domain as well as when opportunities arise during other instructional units in which the concepts of the particular domain are used.

The development of a domain-based assessment program involves creation of numerous opportunities for informal and formal assessment that involve mathematical reasoning at increasingly complex levels and that document growth over time.

Informal Assessment. The assessment program should capitalize on the information that teachers gather through informal assessment methods during instruction rather than assessing only written responses on assignments, quizzes, or tests. Items embedded in instructional materials can be used to focus classroom assessment. The instructional goals of selected items might vary in focus from more narrow levels (e.g., whether students appropriately apply a particular mathematical tool, such as a fraction bar) to a broader application of concepts and reasoning in solving nonroutine problems. Observation of students as they work can provide opportunities for teachers to learn about students' understanding of mathematical concepts, thought processes, and solution strategies. Classroom discussions offer numerous opportunities for teachers to gather information about students' reasoning. Questions can be posed that involve students in seeking patterns, stating and testing conjectures, and moving toward generalizations. The information gained through such informal assessment can enhance instructional decisions. Teachers might, for example, change a familiar problem slightly to stimulate thinking, approach instruction on a concept in a different way, encourage students to make connections among mathematical ideas, or ask students to articulate their thinking in oral or written presentations.

Formal Assessment. Items for formal (end-of-unit or end-of-grade-level) assessments can be selected or created in ways that enable students to demonstrate what they know and can do rather than determine what they do not know. End-of-unit assessments ask students to select and apply appropriate mathematical tools for problem situations that are frequently set in contexts different from those posed during instructional units. Such items provide teachers opportunities to monitor the flexibility with which students use their knowledge of the same concept in a variety of contexts. Also, because end-of-unit assessments are often completed without inter-

action between students or between students and teacher, the items offer opportunities for students to explain and communicate, in writing, their own solutions and thinking processes.

End-of-grade level assessments offer a different perspective about students' developing mathematical knowledge. Because they do not rely on direct connections to particular instructional units, end-of-grade assessments can provide specific evidence of students' increased and more powerful use of concepts, procedures, reasoning, and abilities at the end of a yearlong period of study. Although in theory all domains should be assessed at the end of a school year, no single assessment can contain the quantity and quality of assessment items needed to assess all concepts and procedures. As a result, items on end-of-grade-level assessments are generally designed to provide evidence about the depth of students' understanding related to some of the more important domains addressed in the curriculum. These assessment items present especially good opportunities to assess thinking that involves more than one domain as well as students' abilities to make mathematics their own.

Growth Over Time. Because understanding in a domain is acquired over a long period of time, assessment should reflect the continued development of understanding. Growth in understanding is a gradual process in which the student progresses and regresses in abilities to use the concepts in a domain. Assessment, then, involves the creation of both informal and formal assessments that require attention to relationships the student has constructed at particular points in time, the variance in these relationships over time, the use of knowledge in the domain as resources in approaching novel situations, and the complexity of reasoning applied by the student in the domain. The evidence gathered can be used to inform instructional decisions that further promote learning with understanding, for example, to encourage connections among the concepts of the domain or to provide different contexts for the application of these concepts.

Attention to Levels of Reasoning

As understanding in a particular domain grows, the concepts and procedures of the domain are used in more powerful ways. A domain-based approach to assessment of knowledge in the domain therefore includes attention to the levels of reasoning students apply in using the ideas of the domain. Assessment opportunities involve students in using the mental activities described in Carpenter and Lehrer (chapter 2, this volume) while providing attention to increasingly complex levels of reasoning: reproduction, connections, and analysis.

Reproduction. Mental activities involving reproduction include recall of facts and definitions and routine, efficient application of standard procedures such as performing a particular calculation, solving an equation, or constructing a graph. Assessment items that elicit reproduction are frequently presented without connections to real or imaginable situations in such formats as fill-in-the-blank or short answer. Because basic facts, definitions, and procedures are also used in solutions of complex problems, they can be assessed in student responses for complex problems as well.

An approach to assessment that includes only items that elicit reproduction limits the assessment of knowledge in a particular domain because these items cannot assess the extent to which a student uses the resources in the domain in mathematical reasoning and communication. Assessment items that involve connections and analysis present situations for students to think about and apply mathematics in new situations and provide opportunities for students to develop deeper understanding of mathematical concepts and procedures in a domain.

Connections. Making connections among mathematical ideas involves connections within and across mathematical domains, integration of information, and decisions about the appropriate mathematical tools needed to solve nonroutine problems. Connections among mathematical ideas are formed as students reflect on their existing knowledge in light of new information or new situations. In making connections, students construct relationships between new ideas and existing knowledge. Students also construct relationships as they extend and apply mathematical ideas already understood. Assessment items that elicit connections, therefore, engage students in three of the mental activities that promote learning with understanding and often include articulation of reasoning as well. Such assessment items are frequently set in real or imaginable contexts and, although there is generally only one answer to each problem, can be solved in multiple ways. Student responses, therefore, can show qualitative differences in approach and understanding.

Analysis. Mathematical thinking at complex levels in mathematics includes interpretation, analysis, and mathematical argumentation; development of the student's own models and strategies; and generalization. Often situated in real or imaginable contexts, assessment items that elicit analysis are open-ended and require that students determine appropriate strategies and select appropriate mathematical tools for the solution. Because students often develop innovative solutions for these items, their responses should include all assumptions and reasoning used and the mathematical arguments that support their conclusions. Assessment items that elicit analysis engage students in all mental activities that promote learning with under-

standing. Students must search their existing mathematical knowledge to make sense of the new situation. This involves reflection, construction of relationships, extension and application of mathematical knowledge, and solutions that involve articulation of mathematical ideas. Because assessment items that elicit analysis require students to determine the mathematical ideas and tools needed for appropriate solutions, they are especially useful for assessing the fifth mental activity, making mathematics one's own.

Summary

Understanding in a domain involves the use of concepts, procedures, and strategies in that domain as resources in thinking about and solving novel problem situations. Assessing that understanding will involve multidimensional approaches that make use of both informal and formal assessment, that examine growth of student understanding in a domain over time, and that pay particular attention to levels of reasoning. In the section that follows, we examine specific assessment items and point out the ways in which student responses demonstrate the mental abilities that promote learning with understanding. We then explore ways in which the evidence of students' knowledge produced in responses to these assessments can, in turn, inform instructional decisions for teaching with understanding.

EXAMPLES OF ASSESSMENT ITEMS

In this section, we discuss a variety of assessment items that can be used with the domain-based approach to assessment outlined previously. We present a series of examples from classrooms that show evidence of students' mathematical knowledge as they engage in the mental activities that promote learning with understanding at increasingly complex levels. In these classrooms, teachers are implementing *Mathematics in Context* (MiC), a comprehensive curriculum for Grades 5 through 8 funded by the National Science Foundation. The curriculum includes 40 instructional units (10 per grade level) organized into four domains, each encompassing related mathematical subdomains:

- Number: whole numbers, common fractions, ratio, decimal fractions, percents, and integers;
- Algebra: expressions about variation, graphs, and formulas for patterns and functions;
- Geometry: measurement, spatial visualization, synthetic geometry, and coordinate and transformational geometry; and
- Statistics and probability: data visualization, chance, distribution and variability, and quantification of expectations.

Each unit includes activities that move students toward making connections among subdomains in a particular domain and making connections among domains.

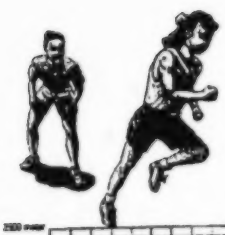
The assessment program for MiC was designed to make the goals of mathematics instruction operational. Teachers are encouraged to use a variety of assessment methods to gather multidimensional information about students' thinking processes, solution strategies, and mathematical knowledge. Assessments include ways for teachers to examine students' abilities to use basic skills; mathematize situations (i.e., extract the mathematics in a situation and use that mathematics to solve problems); reflect on mathematical strategies, discuss results, and formulate arguments that support their thinking; communicate using mathematical vocabulary, diagrams, symbols, and models; and form generalizations. In the example assessment items described next, students are asked to *construct relationships* between text and visual representations and to *extend* and *apply* their mathematical knowledge in making sense of the information provided. In explaining their work to the teacher and in participating in discussions, they are expected to *reflect* on their strategies and use of mathematics in order to *articulate* a mathematical explanation. By thinking about and applying mathematics in the context of a real or imaginable problem, students make strides toward *making mathematics their own.* Their strategies, solutions, and later discussions (both one-on-one and whole class) provide viable evidence of students' understanding of these concepts.

Informal Assessments

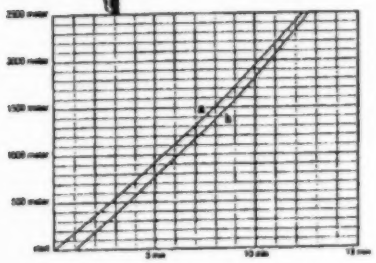
Although assessment of reproduction is often completed during informal assessment, in the sections that follow, we examine items that provide opportunities for students to engage in the five mental activities and assess reasoning at the levels of connections and analysis.

Extension and Application of Knowledge. In Fig. 9.1a, we present a problem from the sixth-grade MiC algebra unit Keeping on Track, in which students explore the connection between individual data points and the continuous nature of line graphs, interpret line graphs by examining both global features (such as regularity and slope) and point-specific information, and write stories that explain line graphs in relation to given contexts. As an assessment embedded in instructional materials, the problem allows students to extend and apply their knowledge of algebra within the subdomain of patterns and functions. To solve the problem, students must integrate their knowledge of minimum and maximum values, slope, and variance with information they have been given in text and graphical representation. The problem, therefore, provides a situation in which students make connections among mathematical ideas.

(a)



Cecilia and her little sister Jasmine are going to race each other. Cecilia decides to make the race fair. She waits until Jasmine has reached the 200 meter mark before she starts to run. Below you see the graph of their race.



10. a. Identify the names of runners a and b. How do you know?

b. Who won the race and how far behind was the other person when the winner crossed the finish line?

c. Who was running faster in this race? How do you know?

(b)

Christina's Response

(A) I think A is Jasmine, because it said that she started first, and A is exactly on the starting line. The book said Cecilia started 200 meters after, so that's the same as the graph. B is Cecilia.

(B) Jasmine won, and Cecilia was about 3 meters behind.

(C) Cecilia was running faster because when they started, she was 200 meters behind, and when they ended, she was only a little behind. Cecilia was able to catch up although she got a later start.

Julio's Response

(A) A is Jasmine because A started first and it said above that Cecilia gave her sister a head start.

(B) A won the race by 20 seconds.

(C) Cecilia was because she started one minute behind and ended up losing by 20 seconds.

FIG. 9.1. Assessment item from Keeping on Track. (a) The problem. (b) Student responses. Used with permission of Encyclopaedia Britannica Educational Corporation.

The responses from two students, Christina and Julio, are shown in Fig. 9.1b. Christina formed correct conclusions: Although Jasmine (the first runner) won the race, Cecilia (the second runner) ran faster. In Part A, Christina pointed out the ways in which the graph verified statements in the text. In her response to Part B, Christina noted that Cecilia was about 3 meters behind the winner rather than about 100 meters behind. Because Part B required an interpretation of specific information in the graph within the context of the problem, to further assess Christina's thinking and knowledge in algebra, the teacher might ask questions to ascertain Christina's strategy for determining the 3-meter difference and scaffold other questions to assist her in interpreting the graph correctly. The response to Part C, although correct, lacked an emphasis on mathematics, making it difficult for the teacher to assess the extension and application of Christina's knowledge of graphs. A more thorough response might have included the distance Cecilia gained on Jasmine during the race, an examination of the starting and finishing times, or an interpretation related to the differences in the slopes of the lines.

Julio also reached the same (correct) conclusion to Part A. His response, although correct, lacked a clear explanation of the connection between the graph and the text. Again, Julio's response to teacher questioning about his reasoning could provide evidence of understanding not available from his written response. In Part B, Julio chose to show the differences in finishing in terms of seconds (rather than meters), demonstrating a qualitatively different, but equally appropriate and accurate, interpretation of Part B. In his response to Part C, Julio used information from the graph to make an interpretation that supported his conclusion. Julio demonstrated his ability to extend and apply his knowledge of algebra in his responses to Parts B and C.

Classroom discussions can provide substantial evidence about students' developing knowledge and understanding in a mathematical domain. The student responses described do provide information about the students' abilities to obtain information about particular points and reason about global information in the graph. But the differences in these students' responses can also be brought out in whole-class discussions, during which students think about the problem situation from another student's perspective and ask questions that clarify various strategies. Expectations for thorough explanations that emphasize mathematics can then be fostered during such dialogue.

Reflection. The assessment item discussed here ("Describe what a shadow is. How are shadows and blind spots similar?") is from the MiC seventh-grade unit Looking at an Angle and was designed to engage students in reflection on geometric ideas in the subdomain of spatial orientation. Throughout the section preceding this problem, students explore

the formation of a blind spot (or shadow) when a vision line (or the sun's rays) hits an opaque object and use a vision line (or representative light ray) as the hypotenuse of a right triangle in side-view representations. Through their explorations, students come to realize that differences in the type of light source (an artificial light source or the sun), the angle of the vision line (or the sun's rays), and the height of the given objects produce differences in the size of blind spots (or shadow lengths). To complete the item, students must consider the effects of the light source and the object blocking the view on the size and shape of the shadow or blind spot and then make inferences about the similarity of shadows and blind spots. In this item, students reflect on the mathematical activities engaged in during instruction and use analysis to mathematize the problem situation, develop their own solution strategies, and explain the mathematics in written form. They also decide whether to use diagrams or models to support their reasoning and–or assist their peers and teacher to understand the mathematics they are using to solve the problem.

Construction of Relationships. The problem shown in Fig. 9.2a is from the MiC sixth-grade unit Fraction Times and is designed to engage students in constructing relationships between number subdomains (fractions and percents) and across mathematical domains (number and statistics). Students interpret the visual representation (a pie chart) of the problem information, relate the (children's) statements to that representation, and compare those statements using their knowledge of fractions and percents. In so doing, they analyze and integrate the information given and apply their knowledge of number subdomains (fractions and percents) and statistics (analysis of representations of data). In each of the responses in Fig. 9.2b, students demonstrated understanding of the pie chart by accurately evaluating the children's statements about the fraction of people who were against construction of new nuclear power plants.

The written responses, however, might not have yielded enough information for the teacher to assess students' construction of relationships among mathematical ideas. If other evidence from classroom interactions is unavailable for these students, the teacher might further assess the relationships they are constructing between number subdomains. For example, the teacher might ask Students C and D how they determined that $\frac{2}{3}$ is about 66%. Their responses might indicate that this is a memorized fact or that they had used a calculator to convert the $\frac{2}{3}$ to an equivalent decimal representation. Other students might have used a number sense strategy such as the one shown in Fig. 9.3a. Still others might have used a visual tool, such as the fraction–percent bar in Fig. 9.3b, to support their reasoning. Further questions, therefore, could provide more evidence of the relationships students were constructing between fraction and percent representations of numbers.

(a)

PIE-CHART METER
More Nuclear Plants?
Every Thursday, Fraction Times features a special page for children aged 8 to 14. The page includes articles sent in by children. Mary, Juan, Karen, and Linda decide to write an article about nuclear energy. As they are researching facts, they come across a pie chart illustrating different views on building new nuclear plants. The children discuss the chart.
Don't Know
In Favor of Building New Nuclear Plants
Against Building New Nuclear Plants
MOST OF THE PEOPLE ARE AGAINST BUILDING NEW NUCLEAR PLANTS.
YES. ABOUT TWO-THIRDS OF THE PEOPLE ARE AGAINST BUILDING MORE.
JUST OVER 60% ARE AGAINST BUILDING MORE.
IT LOOKS TO ME AS IF ALMOST 70% OF THE PEOPLE ARE AGAINST BUILDING NEW PLANTS.
MARY
JUAN
KAREN
LINDA

19. Compare the statements and choose the one that you think best describes the pie chart. Explain your choice.

(b)

A. I think Mary's statement is the best one. Mary said that most of the people are against new nuclear plants and the pie chart shows that 2/3 of the people are against them. 2/3 is most of the people.

B. I think that Juan is the best one because Juan said 2/3 of the people are against it and that's the same thing that the pie chart shows.

C. I think Karen has the best statement. She said just over 60% of the people are against new plants. That is close to the 2/3 shown in the pie chart because 2/3 is about 66%.

D. I think Linda is the best. She said that almost 70% of the people don't want new nuclear plants and the pie chart shows that 2/3 are against new plants. 2/3 is about 66% which is closer to 70% than to the 60% Karen said.

FIG. 9.2. Assessment item from Fraction Times. (a) The problem. (b) Sample student responses adapted from teacher's guide. Used with permission of Encyclopaedia Britannica Educational Corporation.

BEST COPY AVAILABLE

(a)

1/3 of 100 is about 33

2/3 of 100 is double that or about 66

So 2/3 is about 66%

(b)

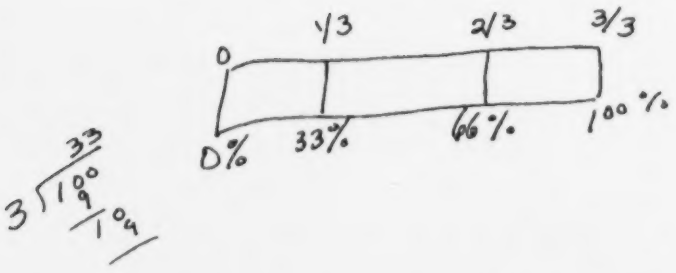


FIG. 9.3. Sample student strategies for determining the relationship between a fraction ($\frac{2}{3}$) and the percent it represents (66%). (a) Number sense strategy. (b). Fraction–percent bar.

Articulation of Relationships and Making Mathematics One's Own. The following problem, from the eighth-grade MiC unit Insights Into Data, is designed to engage students in articulating relationships among numerical and graphical representations of data and in articulating conclusions based on the data. Students are asked to write a report about their experiments dealing with the growth of bean sprouts over time. In this assessment item, students make a statistical summary of the data they have gathered. During this process, they raise the types of questions that should be asked when analyzing data sets: "What is the distribution of the data?" "Are there outliers or clusters?" "Which plot is most useful for initially picturing the data? For comparing data sets from different treatments?" "What statistic is most appropriate for summarizing the data?" These questions allow students to apply their knowledge of single-number statistics (mean, range, median, quartiles, mode) and representations of data (e.g., histogram, box plot, plot over time). For instance, if there is an outlier in the data, students might select the median to summarize the data because the effects of the

outlier do not influence the median. Students might also use knowledge that box plots, although useful in looking at variation among different groups in the experiment, obscure individual data points and decide that a stem-and-leaf plot is more appropriate for developing an initial sense of the distribution of the data.

Ample evidence of students' abilities to articulate the relationships they are discovering can be gathered through listening to student interactions in small groups and observing all phases of their work: planning, selection of numerical and graphical representations, written support for their conclusions, and oral presentation to the class. Although the item elicits analysis, students also make connections when integrating information and making decisions about appropriate statistics and graphical representations and use reproduction when applying technical skills to accurately calculate the statistics and complete representations of the data. In addition to articulating their thinking and conclusions, students move toward making mathematics their own as they explore, represent, interpret, and analyze the data and develop mathematical arguments (statistical summaries, in this case) to support their conclusions.

Formal Assessments

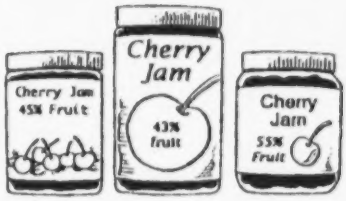
End-of-Unit Assessments. In this section, we examine items from the end-of-unit assessments from Per Sense, a fifth-grade MiC unit. During this number unit, students think about percent as a means of standardization useful in making comparisons between quantities. Algorithms are not emphasized in the unit. Rather, students use mathematical tools such as the percent bar, the ratio table, and benchmark percents (e.g., 1%, 10%, 25%, 50%) to support their thinking, estimation, and calculation. This emphasis gives students opportunities to develop conceptual understanding of percent in realistic problems. The end-of-unit assessment, Jammin', shown in Fig. 9.4, is composed of three questions in the number subdomain of percent (and subdomains of fractions and ratios, if students use the relationships between fractions, ratios, and percent). Because the instructional unit did not include an emphasis on procedures or this particular context for thinking about percents, responses to the questions in this assessment provide evidence about the ways students are moving toward making mathematics their own. Although the end-of-unit assessments provided for MiC units address questions at all three levels of reasoning, these questions assess making connections and analysis.

The first and second questions of this assessment engage students in reflection and articulation of their understanding of the relative nature of percent. In the first question, students are asked to compare the quality of three kinds of jam. The solution requires that students consider the

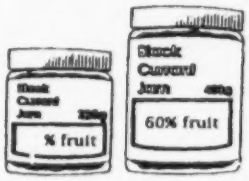
Use additional paper as needed.

Jams can differ in quality. The quality often depends on the percent of fruit that is used for making the jam. The higher the percent, the higher the quality.

1. What can you say about the quality of these three cherry jams?



2. This black currant jam is sold in large and small jars. Someone forgot to put the percent of fruit on the smaller jar. Fill in this missing information. Explain your strategy for finding the percent.



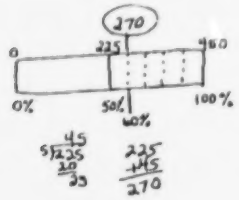
3. How many grams of fruit does this jar contain? Explain how you got your answer.

FIG. 9.4. End-of-unit assessment from Per Sense. Used with permission of Encyclopaedia Britannica.

percent of fruit contained in each jar, not the size of the jars or the amount of fruit (in grams) contained in each jar. The second question, which asks students to find the percent of fruit in the small jar of jam, requires an understanding of the relative nature of percent: Because both jars contain the same recipe of black currant jam, the ratio of fruit to jam (and the percent of fruit in the jam) must be the same in each jar. In the third question, students determine the amount of fruit in a jar of jam when the percent of fruit and the total grams of jam are known. In their solutions, students apply their knowledge of percent to make decisions about appropriate ways to estimate or calculate the amount of fruit.

The students whose solution strategies for this assessment are shown in Fig. 9.5 used four different mathematical tools to support their reasoning and calculation: the percent bar for estimating percents, the 10% (bench-

BEST COPY AVAILABLE

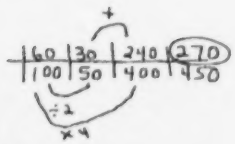


(a)

10% of 450 is 45
60% is 6 x 10%
6 x 45 = 270

(b)

60% is $\frac{60}{100}$



(c)

10% is $\frac{1}{10}$
60% is 6 x 10%
So 60% is $\frac{6}{10}$
$\frac{6}{10}$ x 450 = 270

(d)

FIG. 9.5. Adapted student solution strategies for end-of-unit assessment from Per Sense. (a) Percent bar model. (b) 10% strategy. (c) Ratio table model. (d) Relationship between percents and fractions. Used with permission of Encyclopaedia Britannica.

mark) strategy, the ratio table, and the use of relationships between percents and fractions. The student response in Fig. 9.5a involved the use of a percent bar and number sense to determine that 50% is equivalent to 225 grams and that 10% is equivalent to 45 grams (by dividing 50% into five equal portions). The amount of fruit in the jam (60%) was then calculated by finding the sum of 225 and 45 grams (50% + 10%). Figure 9.5b shows the use of a number-sense strategy based on 10%. The student first determined that 10% of 450 is 45, and, using the knowledge that 60% is 6 times 10%, calculated the answer by taking 6 × 45. Figure 9.5c shows the connection the student was making between percent and ratio. This student wrote a ratio representation for 60% and then used the ratio table to organize calculations with equivalent ratios until a ratio involving 450 grams was found. The response in Fig. 9.5d demonstrates the student's understanding of the relationships between percents and fractions. In this case the student used knowledge that 10% is $\frac{1}{10}$ to write 60% as $\frac{6}{10}$ and then used multiplication with this fraction to calculate the number of grams of fruit contained in the jar of jam.

The questions posed in this assessment allow teachers to assess the flexibility of students' knowledge about percents through their selection and use of various mathematical tools to support accurate conclusions. Teachers might also use student responses to question 3 to look for changes in individual students' use of the concepts and procedures of the domain over the course of the unit. For example, "Did the student use a different strategy on the end-of-unit assessment than on assessments during instruction?" "Did the student apply a method she or he frequently used prior to the assessment, or was there a change to a more efficient or sophisticated solution strategy?" "Did the student make connections between various representations of numbers (percents, ratios, and fractions)?" As teachers reflect on the various pieces of information they gather about the work of individual students during classroom interactions and compare work done on informal assessments with that on end-of-unit assessments, they develop a clearer understanding about the growth in knowledge of particular domains for individual students.

End-of-Grade-Level Assessments. In this section we look at an assessment designed to be given at the end of Grade 5 in the MiC curriculum. A photograph of a giant Ferris wheel taken at a neighborhood festival provides the context for the item. The first of five questions involves whole-number computation requiring reproduction: Given that each of 36 baskets on the Ferris wheel can each hold four people, students are asked to find the number of people who can ride the wheel when it is full. The calculation is straightforward, as student responses indicate: "144—I added 4 up 36 times" or "144 people because 36 × 4 equals 144."

The second question of the assessment directs students' attention to the photograph of the Ferris wheel, noting that it is impossible to count the number of baskets on the Ferris wheel (because of the background in the photograph and the activities at the festival). Students are then asked to describe a way to check if the number of baskets is accurate. Students extend and apply their knowledge of circles (the ways a circle can be subdivided using diameters or radii) and articulate their reasoning. Although fairly easy to solve, this question elicits analysis because it allows students to develop a variety of strategies for solving the problem. Student responses, for example, include: "You can divide the Ferris wheel in 2 [implicit use of the diameter], count the number of baskets on one half of the Ferris wheel, and multiply that by 2," and "Count half the baskets [implicit use of the diameter] and multiply by 2." Some students do find more direct ways of working the problem. The following student response involves only the domain of number: "When the Ferris wheel is full, count how many people come off. Then divide by 4 because that's how many people are in a basket."

In the third question of the assessment, students think about the Ferris wheel in more abstract ways. Visualizing the wheel as a large circle seen from a distance, students imagine they are seated in one of the baskets and are asked to draw the path they travel as their basket moves around the circle. This question elicits analysis because it involves interpretation and mathematization of the problem situation and allows students to develop their own solution strategies. The question provides an opportunity for students to extend their knowledge of geometry (circles) and engages them in making mathematics their own as they mathematize the problem situation.

Two solutions are shown in Fig. 9.6. In the first, a basket is drawn hanging from the center of the circle (Ferris wheel); another circle (of the same radius) is then drawn using the base of this basket as its center. In the second solution, the circle (Ferris wheel) is separated into equal arcs, a basket is drawn at each of the points where the arcs cut the circle, and a new circle is drawn using the bases of the baskets as a guide. These solutions demonstrate quite different yet equally appropriate interpretations and mathematizations of the problem situation and provide evidence of students making mathematics their own.

In the fourth question of the assessment, students are given a diagram of the Ferris wheel along with an enlarged drawing of one of the sections (see Fig. 9.7) and are asked to determine the measure of the angle formed by the bars of the Ferris wheel. Designed for students to make connections among mathematical ideas, students make a decision about the appropriate mathematical tool to use in solving the problem and then apply their knowledge of circles. In this case, students might use a protractor to measure the angle in the wheel, or they might determine the number of divisions

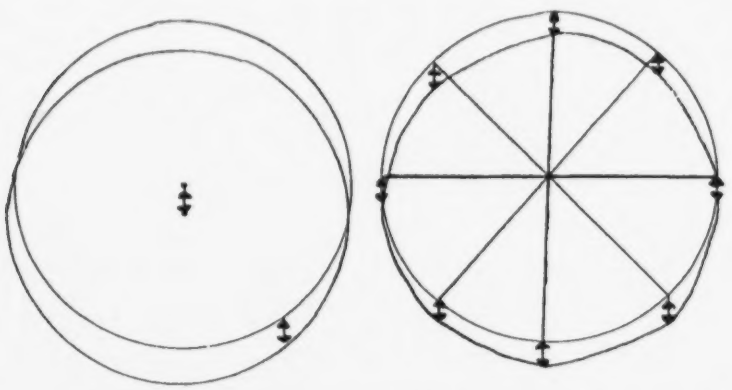


FIG. 9.6. Student solutions for the Ferris wheel item from end-of-Grade-5 assessment. Used with permission of National Center for Research in Mathematical Sciences Education.

In the drawing below you see a model of the front view of the giant wheel.

Part of this front view is drawn below. One basket is attached.

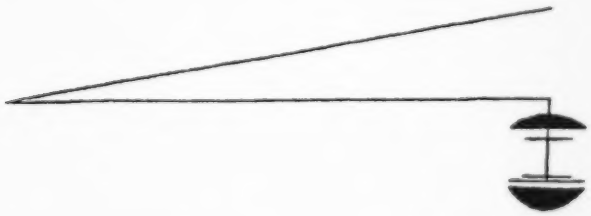


FIG. 9.7. Measuring angle and interpreting scale and ratio in the Ferris wheel item on end-of-Grade-5 assessment. Used with permission of National Center for Research in Mathematical Sciences Education.

made by the bars of the Ferris wheel and divide the total degrees in a circle by that number.

The fifth part of the assessment refers to the enlarged drawing of one of the sections of the Ferris wheel (see again Fig. 9.7): "Kathy says, 'I am sure the bars and the basket in this drawing are not drawn to the same scale.' Explain why her statement is true." The question, designed to elicit analysis, draws on students' interpretation of scale and ratio in the drawing, thus extending and applying their knowledge of number concepts: In constructing a response, students examine the relationship between the size of the basket and the size of the angle formed by the bars. Students typically give responses that do not fully describe their reasoning: "Because if it was true the basket from one bar would be hitting the basket on the bar below it" and "Because the basket is bigger." These responses do, however, provide some insight into the individual student's thinking and can be used in combination with other evidence of the student's knowledge gathered from different sources to assess the student's knowledge in the domain.

The assessment just described provides opportunities for students both to approach problems in a realistic context different from the ones used during instruction and to express their thinking about unfamiliar problem situations related to geometry and ratio. The five questions assess reproduction, connections, and analysis by providing situations for students to perform calculations, make decisions about appropriate mathematical tools, and interpret the situation. Students' responses to these questions can provide ample evidence of their abilities to apply their mathematical knowledge in new situations, to articulate their understanding of the concepts in a domain, and make mathematics their own as they mathematize the problem situations.

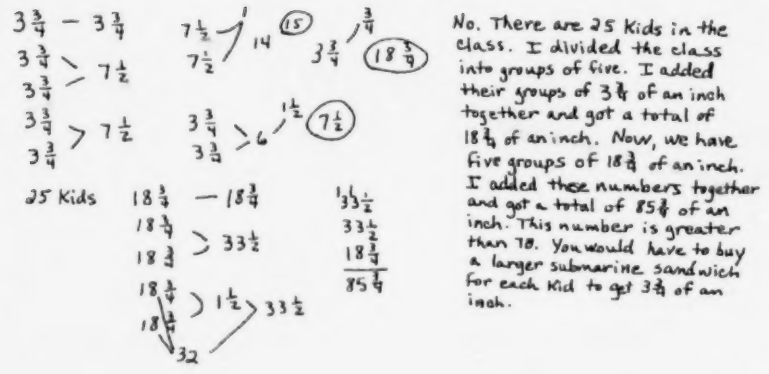
Assessment of Growth Over Time

In order to assess students' understanding of the concepts of a domain, assessment items at multiple points in the instructional sequence focus on students' use of existing knowledge in new situations, new relationships students are forming among mathematical ideas, and the levels of reasoning applied in solving problems. Assessment of rational-number concepts from a domain-based approach, for example, begins with items that build on students' intuitive notions of rational-number concepts and connect this informal knowledge to operations (e.g., ordering, comparison, and calculation) with fractions, decimals, percents, and ratios. Assessment items also include attention to relationships among these representations of numbers, the flexibility of calculation empowered by knowledge of such relationships, the mathematical tools for supporting reasoning with rational-number concepts, and articulation of that reasoning.

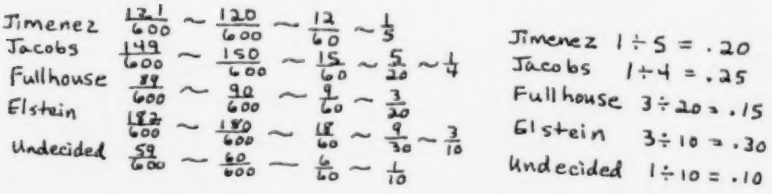
In order to illustrate this concept, we examine the responses of fifth- and sixth-grade students to assessments related to rational numbers. (Also consider in this discussion the evidence gathered about students' understanding of rational numbers shown earlier in this chapter from Fraction Times and Per Sense.) We then conjecture about questions that might guide thinking about growth in mathematical knowledge.

Figure 9.8a shows the response from a fifth-grade student, Jenna, at the end of the first instructional unit related to the number concepts, Some of the Parts. In this unit, students are introduced to addition, subtraction, multiplication, and division with fractions using number sense strategies rather than formal methods of calculation. In this problem, students are asked to determine if one submarine sandwich 78 inches long is enough to serve 25 people, given that each serving is $3\frac{3}{4}$ inches long. In her response, Jenna articulated a convincing argument that details her interpretation of the problem situation and solution of the (division) problem using a repeated-addition strategy. Jenna's solution involves determining the length

(a)


3 3/4 − 3 3/4
3 3/4 > 7 1/2
3 3/4 > 7 1/2
3 3/4 > 7 1/2
7 1/2 + 7 1/2 = 14
3 3/4 + 3 3/4 = 7 1/2
3 3/4 + 3 3/4 > 6
No. There are 25 Kids in the class. I divided the class into groups of five. I added their groups of 3 3/4 of an inch together and got a total of 18 3/4 of an inch. Now, we have five groups of 18 3/4 of an inch. I added these numbers together and got a total of 85 3/4 of an inch. This number is greater than 78. You would have to buy a larger submarine sandwich for each kid to get 3 3/4 of an inch.
25 Kids
18 3/4 − 18 3/4
18 3/4 > 33 1/2
18 3/4 > 1 1/2 > 33 1/2
18 3/4 + 18 3/4 = 32
1 1/2 + 33 1/2
33 1/2
18 3/4
85 1/4

(b)



(c)

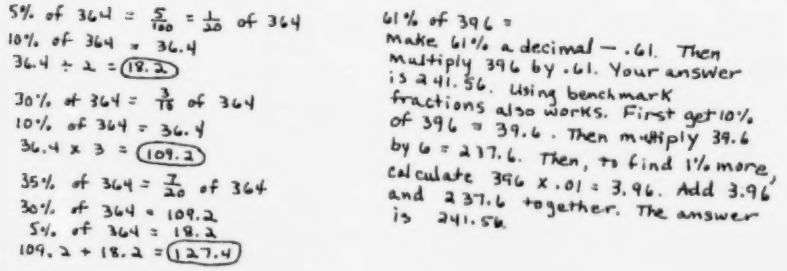

5% of 364 = 5/100 = 1/20 of 364
10% of 364 = 36.4
36.4 ÷ 2 = 18.2
30% of 364 = 3/10 of 364
10% of 364 = 36.4
36.4 x 3 = 109.2
35% of 364 = 7/20 of 364
30% of 364 = 109.2
5% of 364 = 18.2
109.2 + 18.2 = 127.4
61% of 396 =
Make 61% a decimal — .61. Then multiply 396 by .61. Your answer is 241.56. Using benchmark fractions also works. First get 10% of 396 = 39.6. Then multiply 39.6 by 6 = 237.6. Then, to find 1% more, calculate 396 x .01 = 3.96. Add 3.96 and 237.6 together. The answer is 241.56.

FIG. 9.8. Student responses to problems involving rational number. (a) Jenna's response. (b) Mark's response. (c) Marisol's responses. Used with permission of National Center for Research in Mathematical Sciences Education.

BEST COPY AVAILABLE

of sandwich needed for one group of 5 students and finding the sum of sandwich lengths for all 5 groups (25 students) in the class. A minor error in calculation ($18\frac{3}{4} + 18\frac{3}{4} = 33\frac{1}{2}$) led to an inaccurate total sandwich length needed, but because her answer was still larger than the given sandwich (although smaller than the actual length needed), her answer was not affected. After determining the length of sandwich needed for the entire class, Jenna wrote about the solution strategy, providing evidence that she had reflected on the solution strategy before articulating it to the teacher or other class members.

Figure 9.8b shows the response from a sixth-grade student, Mark, for a problem from Fraction Times, a unit taught during the first semester of his school year. The focus of this unit is on formal methods of addition, subtraction, multiplication, and division with fractions and relationships among fractions, percents, ratios, and decimals. In the problem, students are told that the newspaper staff of *Fraction Times* had conducted a survey of 600 people to determine the most popular mayoral candidate. Students are asked to describe with a fraction, then an alternative representation, the portion of votes received by each candidate. Mark's estimation strategy demonstrates the number sense he had developed. He chose to round the numerators either up or down to the nearest 10 and write equivalent fractions for each fraction until it was simplified to an approximation that made sense to him. For example, Mark wrote $\frac{182}{600}$ as $\frac{180}{600}$ and then as equivalent fractions, $\frac{18}{60}$, $\frac{9}{30}$, and $\frac{3}{10}$. Each simplified fraction was then used to express the results in decimal form. Mark showed distinctions between the fractions drawn directly from the data (e.g., $\frac{182}{600}$) and the approximations he used for understanding the data. The responses also demonstrate his knowledge of relationships among fraction, division, and decimal notations for numbers.

Figure 9.8c shows a pair of responses from a sixth-grade student, Marisol, for a problem from More or Less, the second number unit at that grade level. The unit uses everyday situations related to retail business transactions to provide opportunities for students to extend and apply their knowledge of fractions, decimals, and percents. The first set of responses are from a series of eight problems in which students are asked to use benchmark fractions in their calculations. In each of these calculations, Marisol noted the fraction equivalent for the percent, but her solutions involved her knowledge of strategies for calculating with percents. When calculating 5% of 364, Marisol chose to use a strategy based on the fact that 5% is half of 10%. Her calculation of 30% also relied on a strategy using 10% by multiplying 10% of 364 by 3. To determine 35% of 364, Marisol separated 35% into 30% + 5% and, using answers to the previous problems, simply found the sum of these components. The flexibility with which Marisol approached calculations with percents is also demonstrated in her

response to determining 61% of 396. In this set of responses, Marisol clearly articulated two different solution strategies, both of which involved equivalent decimal representations of 61%: a single-step calculation multiplying 396 by 0.61 and a multiple-step calculation using a combination of 10% and 1% strategies. Those responses provide evidence of the relationships Marisol had constructed among fractions, decimals, and percents and the flexibility of calculation gained by applying her knowledge of such relationships.

In assessing the growth over time for individual students, the evidence gathered for each student can be summarized in terms of growth in mathematical knowledge. Suppose the evidence presented previously was gathered for one student over time. The following questions might guide thinking about growth in the domain: "Has the student acquired a variety of approaches and strategies for using mathematical concepts and procedures in the domain?" "How does the student's use of symbols and procedures vary over time?" "In what situations does the student use informal approaches? More formal symbols and procedures?" "Does the student have a solid foundation of informal reasoning to fall back on when faced with more difficult problems in the domain?" This list of general questions could be supplemented with more specific questions about particular domains. With respect to rational-number concepts, additional questions might include, "What relationships is the student constructing among fractions, decimals, percents, and ratios?" "How does the student use equivalent representations of fractions, percents, and decimals to solve problems?" "Which mathematical tool or strategy does the student use to represent and solve problems in which rational numbers are involved?" "Does the student clearly articulate the reasoning used to solve problems with rational numbers?" Such questions can guide later reflections by the teacher on students' growth of understanding and can establish comprehensive evidence of the growth of mathematical knowledge over time.

TEACHER-IDENTIFIED PROBLEMS IN IMPLEMENTING A DOMAIN-BASED ASSESSMENT

Instruction that supports learning with understanding involves more-active roles for students: participation in whole-class or small-group discussion about the central ideas of the domain, the development of their own strategies for solving problems set in real or imaginable situations, and the articulation–communication of their own thinking. Such participation provides multidimensional opportunities for assessment unavailable in traditional mathematics classrooms; yet in order for this information to be useful it must be sought, gathered, and documented. This change in as-

sessment, from solely monitoring the progress of the class toward developing an awareness of the ways individual students reveal their understanding of mathematics, represents a substantial shift in perspective and one that may require professional growth.

Using a domain-based approach to documenting student understanding, teachers shift their thinking away from mastery of skills at the end of an instructional unit to the development of concepts over time. This notion that students need not have complete mastery of concepts before moving on to the next unit (or sometimes grade level) presents a complex dilemma:

> That's always the tightrope you walk as the implementor. As much as we say that, we sort of always want [mastery of concepts] to happen. . . . The question remains, then, *is* there something I can or should do to make this better, or is this just one of those things where we should say, "Well, this is exposure and what happens, happens, and now let's go on to the next thing." You never quite know where to draw those lines. (Shafer, 1996, p. 98)

This teacher's reflection underscores the significance of considering the ways in which mathematics is learned with understanding, the manner in which a particular curriculum addresses this issue, and the instructional sequence of particular concepts (throughout the curriculum) when planning assessment opportunities.

Linkages in a domain in the curriculum and features of problems that permit assessment of developing knowledge of the concepts and procedures of a domain may not be readily apparent to teachers who are starting to implement domain-based curricula:

> During the unit itself, I don't think I was aware of the connections, or perhaps was as aware, as I might be now or next year. . . . Those things come slowly for most of us . . . It's more that I'm still immersed in the day-to-day struggle. But I do know that as we grow, those things become more apparent. (Shafer, 1996, p. 93)

Listing them in teacher guides is not enough, but as teachers teach mathematics for understanding, they become aware of these connections, build on them during instruction, and use them in assessing students' growth in knowledge in a particular domain.

The character of more formal assessments must also change in order to assess students' understanding of relationships among mathematical ideas and their use of mathematical analysis. As teachers focus more on understanding, they begin to recognize and look for differences in levels of reasoning applied in responses to each assessment item. When oppor-

tunities for connections and analysis are not present in publisher-provided materials, it falls to the teacher to write more complex items.

With the change in assessment, teachers face the new, more difficult task of scoring and interpreting items that assess students' abilities in increasingly complex situations—a sharp contrast to traditional mathematics tests in which little variation is expected in student responses, simple scoring rubrics (if any) are used, and grades are easily determined. Richer and more open problems (such as those in domain-based assessment) yield more information about individual students' understanding of skills and concepts than do traditional assessments. As a consequence, responses can be more difficult to interpret than those to (traditional) closed problems and problems not set in contexts. Complicating this is the range of students' abilities to express their own reasoning. Perhaps the most significant step in good assessment practice is the need to understand students' responses, not only by understanding what they actually communicate in writing or orally, but also by understanding what they mean to communicate:

> Naturally, the students should be able to communicate clearly about how they proceeded, and, as such, the communication should be taken into account when grading. On the other hand, however, having insight into a certain topic is not the same as being able to communicate about it. . . . This means that figuring out whether a student has understanding sometimes requires not being distracted by the clumsy wording of the student. (van den Heuvel-Panhuizen, 1995, p. 85)

Looking for reasonable answers is also often more appropriate than looking for correct ones. Again, this implies taking the position of the student in order to make decisions about what a particular answer meant or what the student's reasoning (behind a seemingly wrong answer) might have been. This method of interpreting students' responses allows for more equitable scoring in addition to providing more information about the knowledge of individual students in the domain. Although scoring rubrics and point assignments for partial credit can be composed in advance of grading, unexpected interpretations of the problem, which become evident during analysis of student work, should be added to the categories during the evaluation process.

Another problem in implementing domain-based assessment lies in the limits that teachers' reliance on mental record keeping for accessing information gathered during informal assessments can have on developing an understanding of students' knowledge over a series of units in a domain or across the curriculum. If such information is documented, however, it is available to the teachers not only to refer to and reflect on when assessing student growth in knowledge, but also to share with parents or with other teachers in collaborative discussions. Documentation (whether as notations

on class rosters or as checklists in portfolios) also helps ensure that information is gathered for each student. Confidence in assessment of students' understanding is strengthened when the results of formal assessments can be shown to mesh with the information gathered during informal assessment. (In-depth examinations of informal instructional assessments and documentation of the information gathered are described in several of the readings listed at the end of this chapter.)

Finally, reporting systems that support, not undermine, the new assessment programs will need to be developed. We will need to find ways to create (and to inform audiences about) reliable comprehensive profiles that capture the multidimensional evidence acquired and documented about a student's knowledge and abilities and the processes through which the student's understanding is developing.

CONCLUSION

In this chapter, we have looked more closely at what it might mean to appropriately assess students who are learning mathematics with understanding. Looking for ways to measure the key cognitive activities of understanding requires more than the measurement of basic-skills mastery and rote-memory items provided by standardized tests. If we are to ensure that mathematical understanding is valued, we will need to design and implement ways of showing what such understanding means.

Certainly, assessment results must be reported to the students, parents, and other teachers in forms that are easily understood, and, if we hope to really understand what our students know, in ways that preserve the richness of the information gathered. It will do little good to replace traditional tests (and teaching) with multiple sources related to depth of understanding if this information is collapsed into a single grade or numerical score.

As teachers begin to document specific evidence of a student's knowledge in a domain at particular points in time (and in a student's education) and to record their assessment of the student's growth in knowledge, more comprehensive individual and group profiles of learning, achievement, cognitive processes, and understanding can be developed. Such profiles could provide not only a record of each student's accomplishments and growth but a source of information, far richer than transcripts and standardized scores, on which to base future instructional decisions for each student as well as provide an in-depth portrait of our achievement as a nation.

In this chapter, we have examined one possible assessment program and pointed to the problems and possibilities of its implementation. The program we have described is but a model, but it is a beginning. As we

move toward (and work in) classrooms that promote understanding, we need to rethink, in terms of the next decades of vast, increasingly rapid technological change, what we value as mastery of mathematics—mathematical understanding or rote memorization of formulae and procedures—and retool how we measure and report "achievement."

REFERENCES

Dekker, T. (1997). *Enquiry project: Assessing Realistic Mathematics Education*. Unpublished manuscript, University of Utrecht, The Netherlands.

Shafer, M. C. (1996). *Assessment of student growth in a mathematical domain over time*. Unpublished doctoral dissertation, University of Wisconsin–Madison.

van den Heuvel-Panhuizen, M. (1995). *Developing assessment problems on percentage: An example of developmental research on assessment conducted within the Mathematics in Context project along the lines of Realistic Mathematics Education*. Unpublished manuscript, University of Utrecht, The Netherlands.

FOR FURTHER READING

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22* (3), 170–218.

National Center for Research in Mathematical Sciences Education & Freudenthal Institute (Eds.). (1997–1998). *Mathematics in context*. Chicago: Encyclopaedia Britannica.

Romberg, T. A. (Ed.). (1995). *Reform in school mathematics and authentic assessment*. Albany: State University of New York Press.

BEST COPY AVAILABLE

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

NOTICE

Reproduction Basis

[X] This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

[ ] This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").